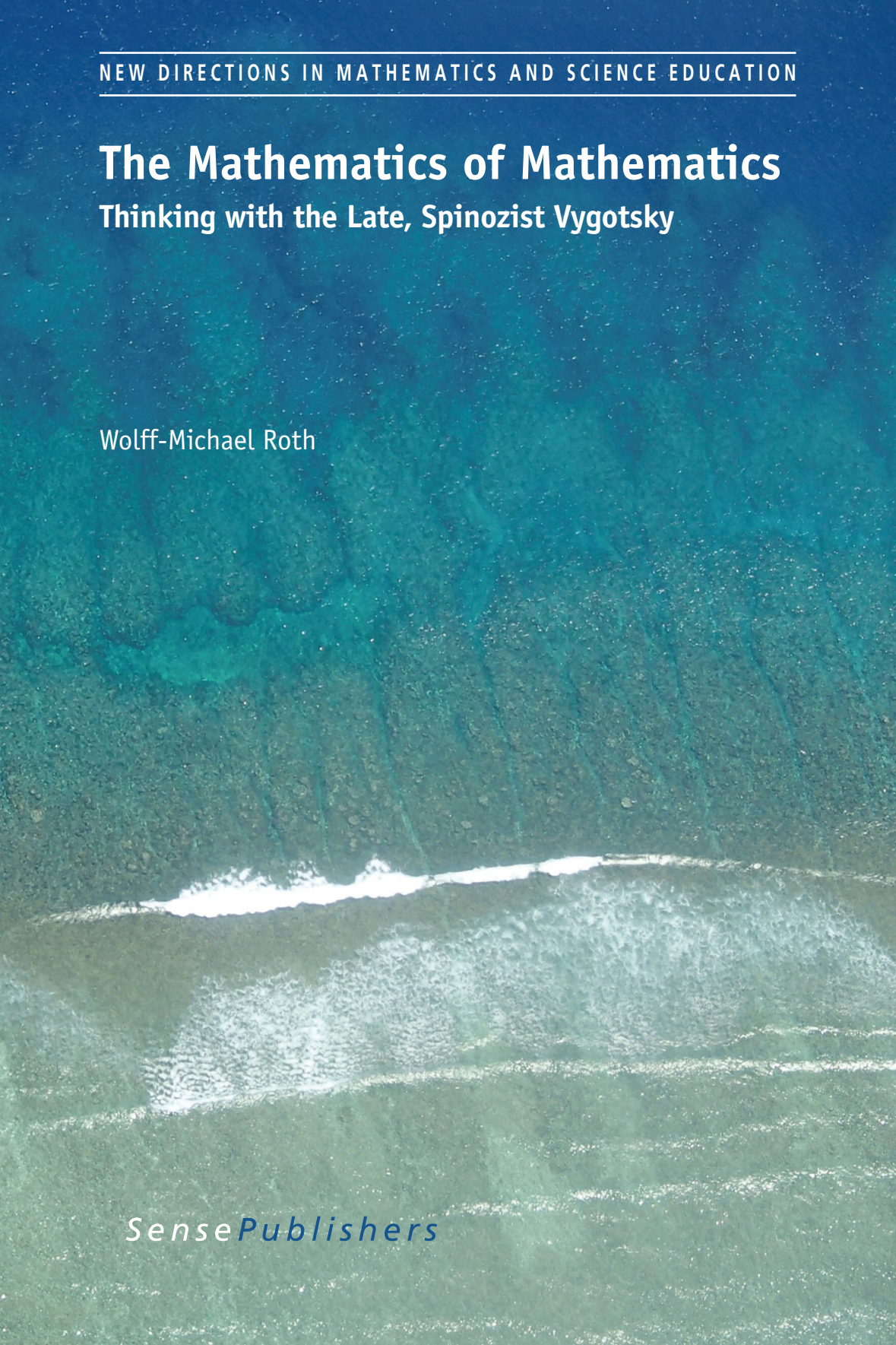
NEW DIRECTIONS IN MATHEMATICS AND SCIENCE EDUCATION

The Mathematics of Mathematics

Thinking with the Late, Spinozist Vygotsky

Wolff-Michael Roth



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The Mathematics of Mathematics

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Scope

Mathematics and science education are in a state of change. Received models of teaching, curriculum, and researching in the two fields are adopting and developing new ways of thinking about how people of all ages know, learn, and develop. The recent literature in both fields includes contributions focusing on issues and using theoretical frames that were unthinkable a decade ago. For example, we see an increase in the use of conceptual and methodological tools from anthropology and semiotics to understand how different forms of knowledge are interconnected, how students learn, how textbooks are written, etcetera. Science and mathematics educators also have turned to issues such as identity and emotion as salient to the way in which people of all ages display and develop knowledge and skills. And they use dialectical or phenomenological approaches to answer ever arising questions about learning and development in science and mathematics.

The purpose of this series is to encourage the publication of books that are close to the cutting edge of both fields. The series aims at becoming a leader in providing refreshing and bold new work—rather than out-of-date reproductions of past states of the art—shaping both fields more than reproducing them, thereby closing the traditional gap that exists between journal articles and books in terms of their salience about what is new. The series is intended not only to foster books concerned with knowing, learning, and teaching in school but also with doing and learning mathematics and science across the whole lifespan (e.g., science in kindergarten; mathematics at work); and it is to be a vehicle for publishing books that fall between the two domains—such as when scientists learn about graphs and graphing as part of their work.

The Mathematics of Mathematics

Thinking with the Late, Spinozist Vygotsky

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Preface

One psychologist more than any other has influenced theories of learning and development: Lev S. Vygotsky. However, recent publications suggest that much of the way in which Vygotsky's work has been taken up does injustice to what the psychologist had actually been writing – to the point of totally misrepresenting the work.[1] During the first decade of the 21st century, Vygotsky's descendants have given Ekaterina Zavershneva access to the family archive.[2] In the process, she uncovered a wealth of unpublished and unheard-of private notes. In these notes that were written near the end of his life, Vygotsky expresses discontent with his own theory, the one most people who read Vygotsky think they are familiar with; and he deems them insufficient, requiring a complete overhaul and revision. In particular, although he had spent much of his scholarly life critiquing and attempting to overcome the Cartesian dualism that is characteristic of psychology then as now he had failed. In the notes, he acknowledges the remnants of Cartesian dualism in his work, including an over-emphasis of the intellectual over affect and the practical. The Cartesianism also characterizes current theoretical approaches, especially in (radical, social) constructivism; but, as philosophers have shown, the specters of Cartesianism exist even in embodiment and enactivist theory.[3]

To overcome these remnants in his own work, Vygotsky turned to the philosopher Baruch Spinoza, who had postulated that there was only *one substance* that has body (Extension) and Thought as attributes. Accordingly, there are not two substances, body and thought (mind), biology and culture, or nature and nurture, but only one substance that manifests itself in different, mutually exclusive ways.

[1] See, e.g., Felix T. Mikhailov, 'The "Other Within" for the Psychologist', *Journal of Russian and East European Psychology* 39 no. 1 (2001): 6–31; and Anton Yasnitsky and René van der Veer, eds., *Revisionist Revolution in Vygotsky Studies* (London: Routledge, 2016).

[2] See, for example, Ekaterina Iu. Zavershneva, 'The Vygotsky Family Archive (1912–1934): New Findings', *Journal of Russian and East European Psychology* 48(1): 14–33; and Ekaterina Iu. Zavershneva, 'The Vygotsky Family Archive: New Findings – Notebooks, Notes, and Scientific Journals of L.S. Vygotsky (1912–1934)', *Journal of Russian and East European Psychology* 48(1): 34–60.

[3] See Maxine Sheets-Johnstone, *The Corporeal Turn: An Interdisciplinary Reader* (Exeter: Imprint Academic, 2009).

This one substance is the thinking body. This thinking body, however, is not the material human body: 'Thought can … only be understood through investigation of its mode of action in the system thinking body–nature as a whole'.[4] Because Vygotsky realized that there are some problems in Spinoza's work, he envisioned reading Spinoza through a Marxian lens generally and through the lens of *The German Ideology*[5] specifically, a book that was published for the first time in Moscow during the final 18 months of Vygotsky's life.

The most fundamental idea in the *Ideology* is the primacy of societal relations to anything that distinguishes humans from other species. Consciousness, the ideal, and the general all are societal in nature – not just as per their origin but also in their very existence. The ideal, such as a mathematical 'abstraction' or a mathematical 'idea', exists in the form of human relations. This insight led me to the title of this book, *The Mathematics of Mathematics*. That is, mathematics is not 'socially constructed' because humans have produced with others (i.e. 'socially') and 'negotiated' some idea. In this way of approaching mathematics, 'the social' is incidental. In any event, any mathematical discovery is ascribed to individuals and, thus, is understood not socially: something like Fermat's Last Theorem is considered to have been an individual construction before it became social. This, however, is not consistent with how Marx or the late Vygotsky thought, where *any higher psychological function* was *a relation with another person*. Therefore, what is mathematical in mathematics is not merely (contingently) social but exists in the form of (universal) societal relations. Children learn mathematics and mathematical forms because they exist in public, that is, because these exist *as* relations in which the children are integral part. Because, according to the *Ideology*, consciousness [Bewußtsein] is conscious being [bewußtes Sein], mathematics becomes individual when the child becomes conscious of the relation with others where the mathematical form exists as joint praxis.

Unfortunately, Vygotsky never got to make these revisions. In his final note prior to entering the hospital where he died only 30 days later, he likened himself to Moses, who had seen the Promised Land but was never allowed to enter it.[6] What was it that Vygotsky saw and wanted to develop for psychology? More importantly, how would research methods and findings look like if he had actually been able to do what he envisioned? What would theories of mathematical thinking look like if we were to follow the lines of thinking apparent from the notes of the late Vygotsky (i.e., during the last 18 months of his life)? This book is designed to show where such a revision takes us in mathematics education.

The revisions that Vygotsky planned included aspects of the theory that continue to be major topics of mathematics education. His ideas about the revision to be undertaken have radical consequences for the way in which mathematics educators think and theorize pertinent issues. Some of the shifts arising from thinking with the Spinozist-Marxian Vygotsky include: (a) mediational nature of the sign (lan-

[4] Evald V. Il'enkov, *Dialectical Logic: Essays on its History and Theory* (Moscow: Progress Publishers, 1977), 52.

[5] Karl Marx, and Friedrich Engels, *Werke Band 3* (Berlin: Dietz, 1978).

[6] Ekaterina Iu. Zavershneva, 'Notebooks', 58.

guage) → semiotic (sense-giving) speech field, (b) meaning → sense, (c) the zone of proximal development → the primacy of the social, (d) thought → unity/identity of intellect, affect, and praxis, (e) thinking → thinking and speaking as two lines of development in communication, and (f) the distinction between intra- and intersubjectivity (inside–outside) → intra-intersubjectivity. These shifts involve a radical revision of what have come to be accepted as fundamental truths in the field of mathematics education, including the process of mediation (L. Radford), the establishment of socio-mathematical norms (P. Cobb), and the constructivist distinction between intrasubjectivity and intersubjectivity (T. Brown).

In this book, I take as a starting point Vygotsky's notes, writings, and presentations from the last 18–24 months of his life; from this starting point, I offer up a theoretical approach to mathematical thinking and learning that differs in some essential ways from other current theories. Episodes mostly from elementary school mathematics classrooms but some also involving scientists are used for developing and exemplifying the theoretical advances and the method of investigation. Published analyses of main contributors in mathematics education are used to show where existing work falls short with respect to meeting the requirements that arise from Vygotsky's revision of his own theory.

A key problem with many English versions of Vygotsky's books and texts has been the problematic nature of the translations, which, in some instances, completely falsified what the scholar actually had written or said. Thus, for example, in many translations, the Russian equivalent of psychic [psixičeskij], has been rendered by means of 'mental' rather than, as Aleksander Luria suggested to Michael Cole, as 'psychological'.[7] The Russian language has two terms that tend to be used where English might employ mental – mental'nyj and duxovnyj. The term 'mental' for psixičeskij is inappropriate because the 'psychic' or 'psychological' includes more than the intellectual-mental. Translating psixičeskij as 'mental' is in direct contradiction with Vygotsky's attempt to do *unit* analysis as opposed to analysis by elements. To deal with this issue, I have in many instances replaced a term in a quotation, after consulting the original text, by means of a more appropriate word and indicate this replacement by means of square brackets. Thus, for example, 'higher [psychological] functions' is used instead of 'higher mental functions' when Vygotsky wrote, in Russian, 'psixičeskie funkcii'.

There are similar issues with the translations of Spinoza's Latin *Ethica* into English. Thus, Spinoza's general definition of the emotions[8] begins in one translation with the words 'The emotion called a passive experience is a confused idea whereby the mind affirms a greater or less force of existence of its body, or part of its body, than was previously the case'[9], whereas another translation renders the

[7] Michael Cole, personal email, June 10, 2015.

[8] The Latin text reads, 'Affectus, qui animi pathema dicitur, est confuse idea, qua mens maiorem sui corporis vel alicuius eius partis existendi vim, quam antea, affirmat …'. See the end of part III of Benedicti de Spinoza, *Ethica: Ordine Geometrico Demonstrata et in Quinque Partes Distincta in Quibus Agitur*, accessed June 29, 2016, http://users.telenet.be/rwmeijer/spinoza/works.htm.

[9] Baruch Spinoza, 'Ethics', in *Complete Works,* trans. Samuel Shirley (Indianapolis: Hackett Publishing, 2002), 319.

same Latin text as: 'Emotion, which is called a passivity of the soul, is a confused idea, whereby the mind affirms concerning its body, or any part thereof, a force for existence [*existendi vis*] greater or less than before'[10]. Having studied seven years of Latin in high school allowed me to check the translations against the original. In each pertinent case, I then chose the one that I thought better or more readably represented the Latin original.

In this book, I use *scholarly* Romanization. Thus, when using Russian words, the scholarly, scientific transliteration produced by the *International Organization for Standardization* was used for the Romanization rather than the transliterations of the American Library Association or the British Standard – as this tends to be done in linguistics and philosophy. I therefore transliterate *perežívanie* [experience] and *obučénie* [teaching | learning], as linguists would do it, rather than perezhivanie and obuchenie, as many educators practice it.

Victoria, BC
December 2016

[10] Benedict de Spinoza, *The Ethics (Ethica Ordine Geomtetrico Demonstrata*), trans. R. H. M. Elwes, available at http://www.gutenberg.org/files/3800/3800-h/3800-h.htm (Project Guttenberg EBook).

1

Vygotsky's Marxist-Spinozist Re/Orientations

> 'This is the final thing I have done in psychology – and I will die at the summit like Moses, having glimpsed the prom[ised] land but without setting foot on it. Farewell, dear creations'.[1]

Late in his life, Vygotsky became highly critical of his own previous work and began to (a) radically reorient his inquiries and, in the process, (b) radically rethink his already continuously developing theory. But his rethinking was only in the beginning when death ended his life. In the introductory quotation, written only weeks prior to his death, Vygotsky likens himself to Moses, who had seen the Promised Land but never could actually reached and set foot on it. What was Vygotsky seeing? More importantly, he was in the process of radically revising his existing work, which means, he is celebrated for work today that he was actually in the process of completely overturning. One entry in his personal notebooks, dated to some time between 1931 and 1933, reads like a programmatic instruction to himself: 'Bring Spinozism to life in Marxist psychology'.[2] Indications of where the thoughts occurring to him were leading are apparent from his personal notes and the final pieces of writing that were published only posthumously – including the first chapter and the final paragraphs of *Thinking and Speech*.[3] These textual pieces contain several quotations and paraphrases from Marx and Engel's *The German Ideology*, a work that was published for the first time in Moscow in 1932 (German) and 1933 (Russian).[4] In this chapter, based on the published personal notes and the

[1] Lev S. Vygotsky, quoted in Ekaterina Iu. Zavershneva, 'The Vygotsky Family Archive: New Findings – Notebooks, Notes, and Scientific Journals of L.S. Vygotsky (1912–1934)', *Journal of Russian and East European Psychology* 48(1), 58.

[2] Lev S. Vygotsky, 'Two fragments of personal notes by L. S. Vygotsky from the Vygotsky family archive', *Journal of Russian and East European Psychology* 48(1) (2010), 93.

[3] Lev S. Vygotsky, 'Thinking and Speech', in *The Collected Works of L. S. Vygotsky. Vol 1: Problems of General Psychology* (New York: Springer, 1987).

[4] Karl Marx, and Friedrich Engels, *Werke Band 3* (Berlin: Dietz, 1978). The quotation marks were removed from the original publication of *Thinking and Speech*, first in subsequent Russian editions, later in the English translations that were not based on the original but on the subsequent, changed versions; there were also parts of the text removed. In the quotations used here, the original of *Thinking and Speech* was used including the omitted text and quotation marks.

last texts and lectures Vygotsky produced, his radically revised orientation to thinking and learning mathematics is described.

Spinoza

> 'Mind and body – are one and the same individual thing, conceived now under the attribute of Thought and now under the attribute of Extension'.[5]

Baruch Spinoza holds a special position not only in the later Vygotsky's thinking but also in the philosophical tradition of materialist dialectics – or dialectical materialism – more generally. Already the German idealist philosopher Georg Wilhelm Friedrich Hegel noted that 'Spinoza is the main viewpoint of modern philosophy: either Spinozism or no philosophy'.[6] Evald V. Il'enkov, a philosopher of materialist dialectics, would later refer to the same Hegel work to emphasize the influence Spinoza had on the subsequent development of dialectical thought. Il'enkov, however, identifies one problem that limits the generality of Spinoza's own work, which exists in the non-coincidence of the logic of Spinoza's thinking with the formal logic underlying the movement in which he developed his system (axioms, theorems, scholia, and proofs).

Foundational Determinations

In contradistinction to René Descartes, whose ontology begins with two basic, incommensurable kinds of substances, body (extension) and soul (mind), Baruch Spinoza begins his unified theory with the definition of *substance* as something conceived only through itself.[7] As a result, an extended body can determine another body, just as thought can influence thought. But no relation is possible across kinds so that the extended body cannot influence thought, just as thought cannot causally affect the body.[8] If body (extension) and thought are related then it is because there is a substance much more original than each and transcending both: 'substance is by nature prior to its affections'.[9] There is one substance, which constitutes Nature as a whole. But this substance cannot be grasped or seen as such, because in thinking, it is reduced to thought; and in perception, it is reduced to

[5] Baruch Spinoza, 'Ethics', in *Complete Works* (Indianapolis, IN: Hackett Publishing, 2002), 259.
[6] Georg Wilhelm Friedrich Hegel, *Werke Band 20* (Frankfurt/M: Suhrkamp, 1979), 162–163.
[7] Spinoza, 'Ethics', 217ff.
[8] This is one of the key points of Lucy Suchman's work on plans and situated actions. Plans do not cause situated actions but rather orient it in a general way. After the fact, they describe, more or less well, whether a person has done what the plans foresaw. See Lucy Suchman, *Human-Machine Reconfigurations: Plans and Situated Actions* (Cambridge: Cambridge University Press, 2007).
[9] Spinoza, 'Ethics', 218.

extension. Extension and thought are attribute*s* of the substance, which manifest themselves in the modes of body and idea, the two affectations of substance of interest. Each mode is something else than the substance, and thus again are substances external to each other that must be conceived through themselves. They are therefore, unable to communicate with each other once conceived in separate ways. This will become the key differences between other approaches seeking to integrate body and mind – see chapter 2 – and the Spinozist-Marxian take that Vygotsky was beginning to articulate in his notes and field-charting texts for the projects to come on affect and consciousness. *Existence*, being, is part of the nature of substance and, therefore, cannot be the distinguishing feature between the different modes.

Substance cannot be a (finite) material thing because it 'would not be substance, if it were an object of experience, i.e., something perceivable; for then it would be a determined, special thing, i.e., a sensual, which is only a finite affection of substance, but not substance'.[10] To understand the relationship between substance and its modes, it is useful to explicate a scientific method articulated only in the 20th century: the documentary method. It is used in the social sciences and humanities in the case of abstract, directly inaccessible phenomena, such as 'worldview' or 'zeitgeist'. None of the associate phenomena is directly accessible to observation. Instead, the phenomenon manifests itself – and is perceived – in documentary evidence. Thus, for example, a painting by Paul Cezanne would be a document of the period 'impressionism' in the field of the fine arts much in the same way as a composition by Claude Debussy would be a document of the equivalent period in music. That is, *impressionism* is not accessible directly but only indirectly, through the different ways that it manifests itself. An analogy may assist in the contradictory ways in which something may appear are ambiguous or bi-stable forms, such as the drawing that appears as duck or rabbit (Fig. 1.1). Each is a manifestation of a whole, a duck–rabbit.

In the same way, the Spinozist *substance* is not accessible other than through its modes. But substance does not merely exist in thought – it could not manifest itself in material form if it were only of the thinking type. It is therefore incomparable, as it does not have an equal. Its reality exists in and of itself. In it, the difference between being and thinking, body (Extension) and mind (Thought), is dissolved. As soon as we begin to think substance *as* thinking/thought or as material, then we have lost the very thing we have started out with. Thus, just as 'light discloses itself as light, substance discloses itself as substance'.[11] Just as it is irrational to ask whether the light I see truly is light rather than darkness, or whether it is particle or wave, the denial of substance would not just take something away from substance but would negate it completely.

This position has consequences for approaching the results of neuropsychological studies, where the relationship of thinking and brain is investigated by means of

[10] Ludwig Feuerbach, 'Spinoza' in *Geschichte der neuern Philosophie von Bacon von Verulam bis Benedict Spinoza* (Ansbach: C. Brügel, 1833), 373.

[11] Feuerbach, 'Spinoza', 376.

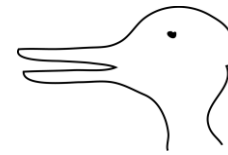

Fig. 1.1 The drawing can be seen as a duck or as a rabbit, but not both at the same time. One may therefore ask, 'Is it a duck or is it a rabbit?' Neither explains itself by the other. A way out of the quandary is to assume some phenomenon that manifests itself in contradictory ways. 'Duck' and 'rabbit' are the analogy of the two modes of one and the same phenomenon (substance)

functional magnetic resonance imaging (fMRI) studies. While the human subject does some task, picking up a cup of coffee, looking at images, or listening to someone else's talk, an image is made of the activity in the brain. These recordings, therefore, represent the biological manifestation of the activity, whereas thinking (understanding, consciousness) is its counterpart in the mode of thought. From the Spinozist position, it is 'impossible either to understand thought through an examination, however exact and thorough, of the spatially geometric changes in the form of which it is expressed within the body of the brain, or, on the contrary, to understand the spatial, geometric changes in the brain tissue from the most detailed consideration of the composition of the ideas existing in the brain'.[12] This is so because, as Spinoza insists, the two forms of movement (change) are the different manifestations of one and the same phenomenon. That phenomenon, thereby, falls outside of the description. Hegel would later launch a very similar critique at the phrenologists of his time, who attempted to correlate bumps on the skull with consciousness of the person.[13] In the same vein, Vygotsky would note that 'it is ridiculous to look for specific centers of higher psychological functions or supreme functions in the cortex … they must be explained not on the basis of *internal organic* relations … but in external terms'.[14]

When two things have nothing in common, such as the two modes of body (extension) and thought, then one cannot be the *cause* of the other. This third proposition of the *Ethics* is counter to almost everything that we are familiar with in current research, for it means that thought cannot cause bodily action, including the bodily movements that bring about speech, and material action cannot cause thought. Some readers may be aware of the research that exhibits the gap between plans, forms of thought, and situated material action[15] – a confirmation of the suitability of the Spinozist conceptualization.

Differing from other ways of conceiving cause and effect, where cause is external to the thing (mode) affected, the Spinozist-Marxian approach takes cause to be *immanent* rather than *transitive* to things. Characteristic of the former way of thinking exists when teachers, what they say or do, somehow are considered to be responsible in one or another way for student learning. Such ways would include

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[12] Evald V. Il'enkov, *Dialectical Logic: Essays on its History and Theory* (Moscow: Progress Publishers, 1977), 36.

[13] Georg Wilhelm Friedrich Hegel, *System der Wissenschaft, Erster Theil, die Phänomenologie des Gesistes* [System of Science, Part I, Phenomenology of Spirit] (Bamberg und Würzburg: J. A. Goebhardt, 1807), 259–286.

[14] Lev S. Vygotsky, 'Concrete Human Psychology', *Soviet Psychology* 27 no. 2 (1989), 59.

[15] See, for instance, Suchman, *Reconfigurations* and Wolff-Michael Roth, 'Radical Uncertainty in Scientific Discovery Work', *Science, Technology & Human Values* 34 (2009): 313–336.

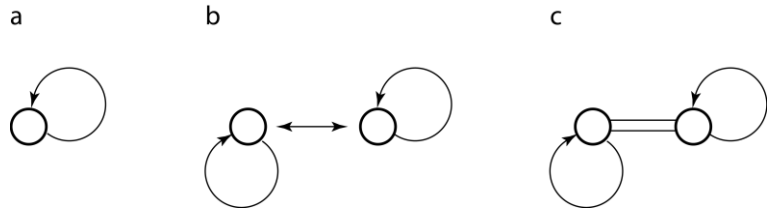

Fig 1.2 **a** In the self-actional model, the Self constructs itself, its knowledge being tested for viability through actions in world forever inaccessible. **b** In the *inter*actional model, two (or more) autonomous Selves relate to each other, each external to the other. **c** In the transactional model, the Selves cannot be specified independently: they are interdependent parts of a whole, each reflecting the whole and, therefore, also the other

thinking of teacher activity as 'scaffolding' students' construction of 'meaning'. Any action inherently is material, so that it does not make sense in a Spinozist take to say that students' thinking changes as a consequence of teacher actions. Instead, the Spinozist take is consistent with transactional approaches, such as those that are articulated by John Dewey or Gregory Bateson. The transactional perspective is opposed to the self-actional and *inter*actional models.[16] In self-actional models of knowing, the individual subject is coiled upon itself, constructing within its mind its own intrasubjective structures (Fig. 1.2a). It is an informationally closed system, which, at best, can test its constructions with its experience in the world. In *inter*actional models, two or more independent Selves (monads) come together to construct the objects of the world and knowledge together, which each then constructs (internally) for itself (Fig. 1.2b). Transactional models are inherently Spinozist, for the philosopher recognized that 'we are passive insofar we are a part of Nature which cannot be conceived independently of other parts'.[17] Thus, any part of the system is given in terms of the system as a whole, that is, it is given in terms of all other parts. In the relation of two Selves, the whole relation determines these Selves, as much the Selves constitute the relation (Fig. 1.2c). For scholars such as Bateson and Dewey, the system investigated includes observers such that it is impossible to decouple observer and observed.

Thought is an attribute of the originary one substance, Nature (God), which, in this, becomes a 'thinking thing'. The two opposites, mind and matter (body) are one in substance. In other words, the notion of substance is present in the contradictory notions and, therefore, is not sublated (abolished). Instead, it is independent of the two such that it can be one or the other, and, in fact, it is both. Substance,

[16] Dewey and Bentley coined the terms self-actional, interactional, and transactional. In a review of the literature on situated cognition, Roth and Jornet used these adjectives to distinguish different models of situated cognition as these have been proposed in different fields concerned with knowing. See John Dewey, and Arthur F. Bentley, 'Knowing and the Known' in *Useful Procedures of Inquiry* edited by Rollo Handy and E. C. Harwood (Great Barrington: Behavioral Research Council, 1999); and Wolff-Michael Roth, and Alfredo Jornet, 'Situated Cognition', *WIREs Cognitive Science* 4 (2013): 463–478.

[17] Spinoza, 'Ethics', 324.

therefore, is a singular plural or, which is the same, a plural singular. At the same time, because any mode has to be considered within one or the other attribute, thought and matter can be distinguished and separated from the thinking thing.

Spinoza notes in Proposition 7 of the *Ethics* part II: '*The order and connection of ideas is the same as the order and connection of things*'.[18] Substance may be considered under the attribute of thinking or under the attribute of extension, which leads us to thinking substance and extended substance. But the two are the same substance, simply considered under the lights of different attributes. Thus, whether Nature is considered 'under the attribute of Extension or under the attribute of Thought or under any other attribute, we find one and the same order, or one and the same connection of causes'.[19] This notion is fundamental to the Spinozist-Marxian approach, where categories are to be developed that indeed correspond to the phenomenon. A psychology based on this approach therefore pursues the genetic, historical reconstruction of its fundamental categories such that these also account for the psychological phenomena and how they are thought about during each epoch. Such historically developed categories may be, and in many cases are, inadequate and confused because they also have an internal logic, as do adequate, clear, and distinct ideas. The historical reconstruction therefore accounts for both the development of real phenomena – i.e. part of the extended substance – and the development of thinking about them – i.e. part of the thinking substance. The reconstruction is required because other approaches – e.g. classical psychology or sociology – operate with everyday concepts that are operationalized within science (see below, section on method). Few scholars seem to be attuned to the fact that the '*the preconstructed is everywhere*' so that the 'first and most pressing scientific priority … would be *to take as one's object the social work of construction of the pre-constructed object*'.[20] This is precisely the point of historically reconstructing psychological categories.

Thought is an attribute of Nature. Its infinite mode is the infinite intellect, whereas its finite mode is the individual mind. The individual human mind therefore is part of the infinite intellect. When there are multiple ideas – concretely realized in different individual minds – then each mind considers the thing partially, one-sidedly, and, thus, inadequately. The 'idea of the body' and 'the body' are one and the same individual thing, considered under the attribute Thought and Extension, respectively. But each of these two attributes, which can be considered only in their own terms, expresses substance only in its own ways, as a particular type: each manifests substance only one-sidedly.

Spinoza proves the proposition that humans exist in the modes of mind and body, united in the person. There is a correlation of the capacity to conceive of things and the variety of states of the body. Some scholars interpret this correspondence as constituting a parallelism of two different, autonomous things, au-

[18] Spinoza, 'Ethics', 247.
[19] Spinoza, 'Ethics', 247.
[20] Pierre Bourdieu, 'The Practice of Reflexive Sociology (The Paris Workshop)', in *An Invitation to Reflexive Sociology* (Chicago: University of Chicago Press, 1992), 235 and 229.

tonomous, because each has to be conceived through itself.[21] The correspondence exists because body and mind are modes of the same single substance, the order of which is expressed identically in each mode. The Spinozist-Marxian take differs from others in how mind and body are approached, that is, through the unity/identity of one substance, which can be understood only through the consideration of that substance, versus through the parallelism of two autonomous systems between which there cannot be any real action.

To explain the relationship between thinking (ideas) and the world, Spinoza develops (a) a list of propositions about the relationship between bodies generally before getting to the relationship between bodies external to the thinking human body and the thinking human body itself and (b) propositions about the relationship between affections of the human body and ideas of these affections. He then proves Proposition II.39: 'Of that which is common and proper to the human body and to any external bodies by which the human body is customarily affected, and which is equally in the part as well as in the whole of any of these bodies, the idea also in the mind will be adequate'.[22] The human thinking body, in contrast to non-thinking bodies, evolves the shape (i.e. trajectory) of its movement in space such that it will conform to the shape, configuration, and position of that other body. That is, the thinking body coordinates the shape of its own movement with the shape of the other body. Take the case of the hands of an expert typist, whose hand and finger movements are such that s/he no longer needs to look at the keyboard to get on paper or monitor whatever text s/he wants. The hand and finger movements have been shaped by the layout of the keyboard. Now we might think that the movement of a compass and a circle also are coordinated, but this is so because of the way in which the compass is constructed. Compared to the compass, the human thinking body builds its shape on many differently formed bodies. It is precisely this feature that distinguishes the thinking body – e.g. of humans – and the non-thinking body – e.g. the compass. This Spinozist definition allows other bodies to be thinking bodies (e.g. animals), though differently and perhaps more limited, such as that of chimpanzees that are capable of making and using (simple) tools.

An interesting aspect in the system of Spinoza is the position on the question of the truth and correctness of ideas and the distinction from error. Because '*the human mind has no knowledge of the body, nor does it know it to exist, except through ideas of the affections by which the body is affected*', '*there is nothing positive in ideas whereby they can be said to be false*'; that is, 'inadequate and confused ideas follow by the same necessity as adequate, or clear and distinct ideas'.[23] Falsity arises from the fragmentary nature of ideas. There are consequently no inadequate or confused ideas unless these are the ideas of a specific individual. True and false ideas are distinguishable based on 'reason' and 'intuition' (i.e. induction and abduction in modern parlance), which are the forms of knowledge of the second and third kind Spinoza identifies; and only knowledge of the first kind,

[21] See, e.g., the discussion of the body–mind (parallelism) by Gilles Deleuze, *Spinoza: Practical Philosophy* (San Francisco, CA: City Light Books, 1988), 86–91.
[22] Spinoza, 'Ethics', 265.
[23] Spinoza, 'Ethics', 258, 263, 264.

knowledge from casual experience and symbols, is the origin of falsity. In this Spinozist take, therefore, 'the erring man also acted in strict accordance with a thing's form, but the question was what the thing was'.[24] This aspect of the theory should lead us to reconsider children's ideas in mathematics, which are not because this or that child thinks incorrectly, but because they are integral aspect of the symbols they find in their environment. The errors need to be situated in the cultural context rather than sought in the (incorrect wiring, structures of the) mind.

Implicit in the Spinozist position is the fact that the body, considered as a body, cannot have (mental) schemas of its own (future) actions that are somehow located within the body generally and the mind specifically. For (the movement of) thinking to be appropriate, the only requirement that needs to be fulfilled is that of the thinking body to act in accordance with any present situation, its forms and arrangements. Thus, '*the idea of any mode wherein the human body is affected by external bodies must involve the nature of the human body together with the nature of the external body*'.[25] This, indeed, can be considered to be the function that was selected during the evolution of the human species because it allowed the organism to act appropriately according to the present requirements. In this situation, therefore, the structure of the thinking body, including that of the brain, tells us little about what the thinking body actually does. This is so because the condition to which it adapts, the 'cause' of its action, generally lies outside of the thinking body. From a Spinozist position it is therefore 'necessary to elucidate and discover in the thinking thing those very structural features that enable it to perform its specific function, i.e. to act, not according to the scheme of its own structure but according to the scheme and location of all other things, including its own body'.[26] That is, this position requires us to investigate the real system, thinking body in its context within which it does and thinks, qua organ of doing and thinking. 'Thought can therefore only be understood through investigation of its mode of action in the system thinking body–nature as a whole'.[27] Readers certainly recognize this as the underpinning of the entire research field known under the label 'situated cognition'. For Spinoza, then, thought is not found in any single body, or system of bodies, but only in Nature considered in its entirety. *Nature* 'is the efficient cause of all things that can come within the scope of the infinite intellect', generating, among others, partial forms of itself realized in the human mind [that] perceives a thing partially and inadequately'.[28]

The thinking body creates an adequate idea of the forms of the objects by creating an adequate idea of itself, that is to say, by creating an adequate idea of the form of its own movement along the surface of external objects. As shown in the preceding typewriter example *the movements of the hands have taken the form of the keyboard, they have the same contour*. Research on eye movements during perception shows that the eyes are indeed 'following' the outline of the object (Fig.

[24] Il'enkov, *Dialectical Logic*, 58.
[25] Spinoza, 'Ethics', 256.
[26] Il'enkov, *Dialectical Logic*, 51.
[27] Il'enkov, *Dialectical Logic*, 52.
[28] Spinoza, 'Ethics', 227, 250.

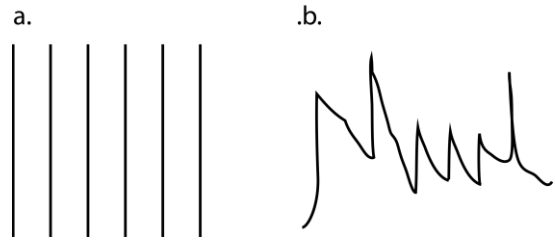

Fig. 1.3 The Spinozist take is consistent with research on eye movement. Our eyes are in constant movement, and it is this movement that allows us to see and know anything at all. If a person is asked to count the number of straight lines in a drawing that includes the set in (a), research has shown that the eyes move in ways similar to (b), i.e., by 'following' each line through appropriate fixations

1.3). It is from the *movement* of the eyes 'around' the object that we come to know. Relatively recent neuroscientific research shows that objects and space are cognized by means of motor neurons that are active when the body moves. That is, we do not see because the world is represented on the retina in the way a photograph is made on a photographic medium – traditional film or electronic charge-coupled device in digital cameras – but because 'movements carve progressively out working space from undifferentiated visual information'.[29] Watching a video clip of someone being disgusted activates similar parts of the brain as when the same person smells a disgusting odor.

But if we recognize a «line» (the material thing) as 'a line' (the 'concept') because our eyes move in a particular way, then the work of my eyes is like that of yours. The same things have shaped their movements in the same way. The social in perception, therefore, arises from the things, which give themselves to all of us and thereby shape the movements of our senses, *not from our individual constructions*!

From the Spinozist position, it is 'in possessing consciousness of my own state (actions along the shape of some contour or other), [that] I thus also possess a quite exact awareness (adequate idea) of the shape of the external body'.[30] Therefore, there is a positive relation between the number of actions a body can perform and its thinking. Spinoza concludes that 'the mind is more capable of perceiving more things adequately in proportion as its body has more things in common with other bodies'.[31] The active aspect of perception also is apparent in the experiences with perceptual puzzles, as the duck–rabbit above (Fig. 1.1), where we might see a drawing initially only under one aspect (perceive one attribute) but where, once shown or encouraged to see something else, suddenly another aspect appears. From that moment on, we will have no trouble seeing one or the other aspect – the eyes have learned to move such that one or the other aspect appears in our conscious perception.

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[29] Giacomo Rizzolatti, Luciano Fadiga, Leonardo Fogassi, and Vittorio Gallese, 'The Space Around Us', *Science* 277 (1997): 190–191.
[30] Il'enkov, *Dialectical Logic*, 70.
[31] Spinoza, 'Ethics', 266.

Research in science classrooms on the birth of signs is consistent with rather than contradicting this take. Thus, students asked to explain the results of their investigations initially reproduced the movements required to do the investigations, then used the same hand/arm movements but symbolically, before eventually developing verbal descriptions that took over from and did the communicative work of the earlier movements.[32]

This way of approaching the issue offers opportunities to overcome Cartesian dualism, because thinking arises from acting along the contours of the social and material world. The actions of the thinking body have been shaped in their relations with the world. Consider again the finger and hand movement of the expert typist, who no longer needs to look at the keyboard to re/produce a text on the computer monitor. These movements have been shaped by the geometrical configuration of the (QWERTY or other) keyboard such that the hands type without requiring the guidance from the perception of the keys. Those same hands, however, do many other, quite different things equally well, such as handling knife and fork while eating, holding a pen while writing, or tying shoelaces. Thus, the competence related to typewriting – here standing for any cultural practice and thinking – are not found in the structures of the body, hand or the brain, but in the observable composition of its actions in real, objective activity with and among other bodies (e.g. hands and keyboard). Thought is not found in the structure of the brain. Instead, to find it one has to investigate 'the "inorganic body of man", the "anatomy and physiology" of the world of his culture, the world of the "things" that he produces and reproduces in his activity'.[33] The example of the QWERTY keyboard – which is not a 'natural form' but one created by humans – also shows where the Spinozist position falls short: in failing to recognize that human beings not only are subject to conditions, learning by contemplating the world, but actively change this world to give it new form. These new forms give rise to specifically human, that is, societal forms of consciousness, which is a central point that Karl Marx makes when, in *Das Kapital* (Capital), he develops his theory of (exchange-) value, the *supersensible* pole of the *sensible-supersensible* commodity entering an exchange relation.[34]

The upshot of these considerations is that thinking is not a property of the individual body but of nature as a whole. Moreover, in the Marxian take, the specifics that distinguish humans from other beings are their societal relations and society. Therefore, 'a body of smaller scale and less "structural complexity" will not think. Labour is the process of changing nature by the action of [societal] man, and is the "subject" to which thought belongs as "predicate"'.[35]

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[32] Wolff-Michael Roth, 'The Emergence of Signs in Hands-on Science', in *International Handbook of Semiotics,* edited by Peter Trifonas (Dordrecht: Springer, 2015), 1271–1289.

[33] Il'enkov, *Dialectical Logic*, 74.

[34] Karl Marx and Friedrich Engels, *Werke Band 23: Das Kapital: Kritik der politischen Ökonomie Erster Band Buch I: Der Produktionsprozeß des Kapitals* (Berlin: Dietz, 1962), 85–98.

[35] Il'enkov, *Dialectical Logic*, 74. The original uses the equivalent of 'societal man [obščestvennogo čeloveka]' rather than 'social man' that appears in the English translation. See Evald V. Il'enkov, *Di-*

The Spinozist take, reread through the Feuerbach-Marxian lens, also can be found in late 20th century practice theory, where it is recognized that 'the world is comprehensible, immediately endowed with [sense], because the body, which, thanks to its senses and its brain, has the capacity to be present to what is outside itself, in the world, and to be impressed and durably modified by it, has been protractedly (from the beginning) exposed to its regularities'.[36] It is precisely because we are in, surrounded by, and part of the world that we also comprehend it. That is, whereas 'the world encompasses me, comprehends me as a thing among things … I, as a thing for which there are things, comprehend this world. And I do so (must it be added?) *because* it encompasses and comprehends me'.[37] We acquire practical knowledge and control of the encompassing space because we are materially included in the space, even though we may not notice it expressly and maybe even repress it. As a result, the social and material structures of the world that we inhabit are 'sedimented' as forms of dispositional structures, that is, in the forms of expectations and anticipations characterizing our being in the world. That comprehension arises from the openness of the body to the world, where it may be impressed as any other thing. 'Having the (biological) property of being open to the world, and therefore exposed to the world, and so capable of being conditioned by the world, shaped by the material and cultural conditions of existence in which it is placed from the beginning, it is subject to a process of socialization of which individuation is itself the product, with the singularity of the "self" being fashioned in and by social relations'.[38] Important to this practice-theoretic understanding is the rereading Marx enacts of Spinoza. The latter only recognized the impressions that the world makes on the body, whereas Marx specifically states that consciousness arises in and from the active changes that the thinking body effects in the world. Unsurprisingly, therefore, Bourdieu refers to Marx's 'Theses on Feuerbach' and the distinction that is made there between all preceding materialisms, which recognized only a passive side of comprehension, and articulates the necessity to develop a theory that recognizes the active side by means of which a socialized body produces the very conditions of which it is itself the product. In the end, therefore, the practical sense is inhabited by the world that it inhabits. The practical sense allows humans to act appropriately, *comme il faut,* in the heat of the situation, without having to stop and reflect. The thinking body – having been shaped by and while shaping the world – has developed dispositions that allow it to engage in a world immediately endowed with sense, in correspondence with the field. The dispositions, structured by the active and passive experiences of the world, are structuring in the perceptions of and actions in and toward this world.

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alektičeskaja logika: očerki istorii i theorii [*Dialectical Logic: Essays on its History and Theory*] (Moscow: Izdatels'stvo politi Izdatels'stvoeskoj literatury, 1984), 54.

[36] Pierre Bourdieu, *Méditations pascaliennes* (Paris: Seuil, 1997), 163.

[37] Bourdieu, *Méditations*, 157.

[38] Bourdieu, *Méditations*, 161.

From Vygotsky's Readings of Spinoza

Vygotsky programmatically wrote that he would bring Spinoza to life in Marxist psychology. This was so because the philosopher provided a foundation for a *monist* psychology that was to replace both dualistic and parallelist takes on the mind–body relation. Vygotsky noted that the problem of other approaches in bringing together mind and body ended up creating some form of theory of unity and failed to establish it as theory of identity of the psychological and physical. He found the greatest idea of the philosopher, one that 'liberated all of psychology', in the acknowledgment of the relative power of the soul (Lat. *animus*, soul, nowadays is rendered as 'psyche' or 'mind'). That is, 'the relationship between the soul and the body (life and intellect) are not absolute and immutable, but changeable, relative'.[39] The opportunities that arise from such statements for psychology are studies of the 'changes in the proportionate role of the soul in the life of the body, and of the intellect in life'; and, most importantly, 'everything in Spinoza breaks with mechanical causality, with immobility (there is no development, with disconnectedness, with parallelism and demands an escape beyond their boundaries'.[40]

Vygotsky apparently did not tolerate *inter*action, for parallelism, though inherently false, still constituted a consistent theory. Underlying the historical consideration of the teaching of emotions is the reading that – as different but parallel substances – soul and body could not act upon each other (i.e. *inter*act), because the two manifestations would be external to each other. Vygotsky followed the idea of the underlying unity, which would be articulated and developed in the notion of the thinking body by Il'enkov and Merab Mamardašvili, two materialist dialectic philosophers (see chapter 2). In fact, it would be to Il'enkov to provide a well-grounded philosophical justification of numerous key tenets of Vygotsky's cultural-historical theory and developmental teaching | learning (obučenie).[41]

Spinoza notes that '*the human mind does not perceive any external body as actually existing except through the ideas of affections of its own body*'.[42] We find this same idea, which makes the connection between the material world and thought, in the personal, fragmentary notes of Vygotsky. Thus, following the articulation of thinking and speech as the core problem of psychology, he notes, 'only through this issue can one correlate thinking and brain-functioning, just as only through the brain and its motion (which is what embodies psychic momentum) can one correlate thinking and the law of conservation of energy, thinking and a change in the direction of a moving point without expenditures of energy, thinking and catalyzing processes'.[43] Gestalt psychology is critiqued because it bases itself

[39] Lev S. Vygotsky, in Ekaterina Iu. Zavershneva, 'The Vygotsky Family Archive: New Findings – Notebooks, Notes, and Scientific Journals of L.S. Vygotsky (1912–1934)', *Journal of Russian and East European Psychology* 48(1), 39.

[40] Vygotsky, 'Family Archive', 39.

[41] Vasily V. Davydov, 'Vklad E. V. Ilyenkova v teoretičeskouju psixologiju [E. V. Ilyenkov's contribution to theoretical psychology', *Voprosy Psixologii* (1994: 1): 131–135.

[42] Spinoza, 'Ethics', 261.

[43] Vygotsky, 'Two Fragments', 94.

on two different series of concepts, whereby the structure of the brain is set equal to the structure of inner experience. This, Vygotsky notes, does not bring psychology closer to a materialist understanding of the psyche. Speech, having both material (phonetic) and ideal (semantic) dimensions, is the key to the psychophysical (mind–body) problem. Indeed, being sensible–supersensible in the same way as commodity, and serving human exchanges in the same way, we can think of words as Marx was conceiving of commodity.[44] This then allows us, as Vygotsky notes, to overcome the body–mind dichotomy. In his notes, he relates 'the entire psychophysical problem' to a solution grounded in the reading of *The German Ideology*: 'Cf. Marx: the curse of matter on pure consciousness is moving layers of air, i.e., intercourse with the aid of language, rather than a connection with the brain!'.[45] He emphatically adds the German equivalent of 'very important'.

For Vygotsky, 'the psychophysical problem', 'the question of the spiritual and material in human consciousness', 'consists in the relation between thinking and speech', a problem that manifests itself in 'a parallelistic correlation of previously severed attributes'. [46] Vygotsky is interested in the *real* unity rather than the imaginary one that is generated by means of a correlation of externally related things. The guiding idea is situated in 'Marx: the materiality of consciousness in its link to language', which 'is historical materialism (its concrete principle) in psychology', so that 'thinking and speech are the central problem and the *via regia* of all historical psychology'.[47] Moreover, related to the Spinozist topic of will, he notes that it 'must be derived from … soc. relations internalized and embodied in the activity of the centers [cortex, subcortex] with use of natur., organic subordination (sublated category, actuating mechanism'.[48]

Vygotsky was particularly interested in a Spinozist take on emotions because the philosopher had articulated an idea of the person from a 'peak' standpoint. This idea – which 'is the true idea, because it is in agreement with its object'[49] – would be the guiding one for his Marxist psychology. We observe the previously articulated Spinozist take on the correspondence between categories and natural phenomena. Vygotsky died shortly after having written these notes; and he therefore never could articulate a theory of emotions, leaving only a preparatory text 'The Teaching about Emotions'.[50] One of his students, Alexei N. Leont'ev, achieved such a categorical reconstruction of the psyche, whereby acting, sensing, and emoting came to be irreducible manifestations of life beginning with the simplest of organisms.[51]

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[44] Wolff-Michael Roth, 'A Dialectical Materialist Reading of the Sign', *Semiotica* 160 (2006): 141–171.
[45] Vygotsky, 'Two Fragments', 95.
[46] Vygotsky, 'Two Fragments', 93–94.
[47] Vygotsky, 'Two Fragments', 94. *Via regia* is Latin for 'royal road'.
[48] Lev S. Vygotsky, in Ekaterina Iu. Zavershneva, 'The Vygotsky Family Archive (1912–1934): New Findings', *Journal of Russian and East European Psychology* 48(1), 29.
[49] Vygotsky, 'Family Archive', 41.
[50] Lev. S. Vygotsky, 'The Teaching about Emotions: Historical-Psychological Studies', in *Collected Works Vol. 6* (New York: Plenum, 1999), 71–235.
[51] Alexei N. Leontyev, *Problems of the Development of the Mind* (Moscow: Progress Publishers, 1981).

An important aspect of the relationship between thinking and speech is the recognition that speech is not the externalized result of a finished thought but that thought becomes itself in speaking. Speaking has an important bodily aspect that makes it different from the semantic, ideal aspects (i.e. "meanings") that are attributed to words. As speech unfolds, thinking develops so that the mind grasps its thinking only when speaking is finished. This is actually an old idea, which we may find in Spinoza's Proposition II.23, which states: '*the mind does not know itself except insofar as it perceives ideas of affections of the body*'.[52] Here we need to keep in mind that the affections are the modes themselves. For there to be ideas, affections of the body have to exist, and these ideas of the affections allow mind to know itself. Hence Vygotsky's articulation: thought exists once affection, material speech, has come to conclusion.

An important feature of Spinozist method exists in the way Vygotsky relates 'word meaning', 'thinking', and 'speech'. For Spinoza, the order in two modes of a substance is the same not because these somehow interact but because they are fundamentally united in *substance*. Because the whole is present in all of its parts, therefore, word meaning, if indeed it is a whole, has to be present in its modes (parts), thinking *and* speech. But this whole is different from the parts, which only one-sidedly manifest the whole and, therefore, are lacking in this or that aspect – precisely those that exceed the sum of the parts.

Vygotsky and the Feuerbach-Marx Connection

> 'It is not fortuitous that Spinoza's profound idea only first found true appreciation by the dialectical materialists Marx and Engels'.[53]

Karl Marx and Friedrich Engels took as their starting point for understanding human behavior Spinoza's recognition of thought and extension as two attributes of the same substance – as evidenced in the response that the latter gave to the philosopher Georgi Plekhanov, 'old Spinoza was quite right'.[54] Spinoza already emphasized *thought* as an attribute of substance rather than of the individual mind – which is only a finite mode. For Marx, thought, consciousness, is a *societal* product from the beginning, and remains such as long as humans exist. This idea was initially articulated in the works of the Spinozist philosopher Ludwig Feuerbach. These influenced Marx in his early years, such as when we wrote that '*society* is the complete unity of man with nature – the true resurrection of nature – the accomplished naturalism of man and the accomplished humanism of nature'.[55] The

[52] Spinoza, 'Ethics', 260.
[53] Il'enkov, *Dialectical Logic*, 43.
[54] Georgi V. Plekhanov, *Selected Philosophical Works Vol. 2* (Moscow: Progress Publishers, 1974), 339.
[55] Karl Marx and Friedrich Engels, 'Ökonomisch-philosophische Manuskripte (1844)', in *Werke Band 40* (Berlin: Dietz, 1968), 538.

society-producing relations *among* humans determine the relations between humans and nature, and the relations of humans to nature determine the society-producing relations among each other. A Marxian psychologist later would note, accordingly, that we do not impose human significations upon nature but we disclose nature from these significations.[56] In this statement, we observe the Aristotelian conception of the apophantic nature of the statement, which is to let something show itself to be seen. Abstracted from their connections within the ensemble of relations of society, which also give shape to the ways in which humans relate to nature, human beings would think as little as a brain disconnected from the body of the organism. Thought, the ideal, therefore is not an individual psychological fact but instead a specifically human image of objective reality in human activity, which exists in the form of collective, societal consciousness. The life of the human ancestors, who became human during anthropogenesis, did not start with language in the way we know it today. They used sounds in the way animals use it, as a manifestation and function of activity, such as when the cry is used to spread affect. Marx and Engels exhibit the irremediable connection between the emergence and development of language and the development of human society. This point was not lost on Vygotsky, who notes that in individual development, 'the word did not exist in the beginning. In the beginning was the deed. The formation of the word occurs near the end rather than the beginning of development. The word is the end that crowns the deed'.[57] He thereby captured that children from the beginning of their lives – and even before, while still being in the womb – are exposed to the sonorities of language and then participate in a world structured, among others, by the sound-words that accompany and constitute activity. It is out of being with others in the word that consciousness arises. This precisely is the Marxist take, for 'consciousness [*Bewußtsein*] never can be anything else but conscious being [*bewußtes Sein*]'.[58] Consciousness always follows life, emerges from life, but does not determine life.

Vygotsky was deeply influenced by his reading of *The German Ideology*, which was published in Moscow, first in German (1932) than in Russian (1933), that is, he accessed this work during the final period of his life. This can be seen in his use of direct quotations from and paraphrases of this work – which have been omitted and are unacknowledged in the English translation of *Thinking and Speech* – and in parts of the text that have been excluded in the version most Anglo-Saxon readers know. These excluded quotation marks and text have been included in the following quotation, which constitutes part of his articulation of those aspects that *Thinking and Speech* has not covered but lie at 'the threshold of a problem that is broader, more profound, and still more extraordinary than the problem of thinking'.[59]

[56] Georges Politzer, 'Les fondements de la psychologie: psychologie mythologique et psychologie scientifique', *La Revue de la Psychologie Concrète* 1 (1929), 27.
[57] Vygotsky, 'Thinking and Speech', 285.
[58] Marx and Engels, *Werke Band 3*, 26.
[59] Vygotsky, 'Thinking and Speech', 285.

> 'The consciousness of sensation and thinking are characterized by different modes of reflecting reality. They are different types of consciousness. Therefore, thinking and speech are the key to understanding the nature of human consciousness. But if "language is as ancient as consciousness itself", if "language is consciousness that exists in practice for other people and therefore for myself", when "the curse of matter, the curse of moving layers of air hangs over consciousness from the beginning", then it is not only the development of thought but the development of consciousness as a whole that is connected with the development of the word. Studies consistently demonstrate that the word plays a central role not in the isolated functions but the whole of consciousness. In consciousness, the word is what – in Feuerbach's words – is absolutely impossible for one person but possible for two. The word is the most direct manifestation of the historical nature of human consciousness'.[60]

We note that the psychophysical problem is at the heart of the matter, the fact that language and consciousness are weighted down by the 'curse of matter', an aspect that is completely absent in the modern day concern for 'meaning' that is even less material than language in its various instantiations. The descendance of Vygotsky's position at the end of his life from the works of Marx and Engels is apparent in the following quotation concerning the materiality of human consciousness.

> 'But even from the outset this is not "pure" consciousness. The "mind" is from the outset afflicted with curse of being 'afflicted' with matter, which here makes its appearance in the form of agitated layers of air, sounds, in short, of language. Language is as old as consciousness, language *is* the practical, real consciousness that exists for other men as well, and only therefore does it also exist for me; and language, like consciousness, only arises from the need, the necessity, of intercourse with other men. … Consciousness is at first, of course, merely consciousness concerning the *immediate* sensuous environment and consciousness of the limited connection with other persons and things outside the individual who is becoming conscious of itself; it simultaneously is consciousness of nature, which first confronts men as a completely alien, all-powerful and unassailable force'.[61]

The important aspect of this take is that society and societal relations are a condition for the emergence of consciousness in the form of language, which is an integral part in the material life of society, the real process of labor that is oriented to the satisfaction of basic and extended (not hedonistic, but real, life-preserving) needs. For Marx and Engels, society is that unit that allows us to understand the individual; and, as stated in the Eighth Thesis on Feuerbach, 'all societal life is essentially *practical'*. What is separate in the theories that tend to mysticism, the

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[60] Lev S. Vygotskij, *Myšlenie i reč': psixologičeskie issledovanija* (Moscow: Gosudarstvennoe social'noèskonomičeskoe isdatel'stvo, 1934), 318.

[61] Marx and Engels, *Werke Band 3*, 30–31.

individual and the social, bodily and mind, exist in the true unity of human, societal praxis and in the grasp of this praxis. This grasp is the consciousness of practical life, which operates at two levels. On the one hand, consciousness of relations with other people precisely constitutes the higher psychological functions that Vygotsky eventually would emphasize; and researchers themselves, rather than mystifying these functions, can comprehend the origins and nature of these functions by investigating their appearance anthropologically in real, concrete praxis. This allows comprehending sensuousness as practical activity.

In its foreword to *The German Ideology*, the Institute for Marxism-Leninism of the Central Committee of the Communist Party of the Soviet Union describes the nature of the text to be foundational for a number of social sciences conceived from a Marxist point of view. Thus, the text is foundational because it relates the creation and development of language with the material life of society, human labor. The Institute for Marxism-Leninism notes that Marx and Engels emphasize the irremediable *unity* of language and human thinking and that the truth and reality of thought manifests itself in language (speech): 'Marx and Engels explicate the nature and function of thinking, the intellectual *needs, interests, inclinations and feelings of man*, they show that the decisive causes for their change and development are grounded in material life of society, and they thereby lay the foundation for a Marxist, dialectical-materialist psychology'.[62] In his discussion of the problems of the classical social sciences, Vygotsky points out the separation of the intellectual from the material aspects of language, intellect was abstracted from a whole that also has affect as one of its attributes. His own writings reflect his readings of these passages from Marx and Engels. In the classical approach, thinking was theorized in itself, divorced from life, 'from the living *motives, interests, and inclinations of the thinking person*'.[63] To understand a thinking *person*, however, requires more than considering what happens in the mind. This is so because 'to explain the event we call "thinking", to disclose its effective *cause*, it is necessary to include it in the chain of events *within which it arises of necessity and not fortuitously*. The "beginnings" and the "ends" of this chain are clearly not located within the thinking body at all, but far outside it'.[64]

The concrete human psychology that Vygotsky envisioned conducts deterministic analyses of thinking, which presuppose 'the identification of its *motive force, the needs and interests, incentives and tendencies* that direct the movement of thought in one direction or another'. The motive force of thinking does not lie in thinking itself. Thus, 'it is not thought that thinks: a person thinks'[65], an idea that Vygotsky has taken from Feuerbach, who writes: 'Man thinks, not the "I", not reason'. Philosophy and psychology therefore have to have as their objects the '*real and whole being of the person*', for 'the *reality*, the *subject of reason* is man alone' and '*only man is the true and the real*'.[66] It is in this totality of human existence

[62] Marx and Engels, *Werke Band 3*, x, emphasis added.
[63] Vygotsky, 'Thinking and Speech', 50.
[64] Il'enkov, *Dialectical Logic*, 37.
[65] Vygotsky, 'Concrete Human Psychology', 65.
[66] Ludwig Feuerbach, *Sämtliche Werke, Zweiter Band* (Leipzig: Otto Wigand, 1846), 339.

that we can find the unity of thinking and being, a fact that can be grasped only when human beings are considered as the foundation, the subject of this unity. Thought is not severed from the whole person only when we considered thinking not as a *subject for itself* but predicate of a real person.

Vygotsky's theoretical move thereby becomes consistent with the Marx's Second Thesis on Feuerbach, which states that the question concerning the concrete truth and reality of human thinking is not a theoretical but a practical one. It is therefore 'ridiculous to look for specific centers of higher functions in the cortex … they must be explained … in external terms, on the basis of the fact that man controls the activity of his brain from without through stimuli'.[67] It was indeed not Marx (and Engels) who realized that the truth of thinking is a practical question, but something that before them the philosopher Feuerbach had already noted. The latter already suggested that mind, consciousness, and rationality are not fixed attributes, not properties inherent to humans but only exist in and through activity in which they have a concrete practical function. As a result, the question about the nature of being always is and remains a practical question; and, reflexively, our being participates in this question. Something thought theoretically or talked about does not have to really exist, does not have to be a real, material being – a unicorn as a living animal is not part of our material reality just because we can paint one or describe it in words. This is so because in everyday praxis, human beings have to prove the truth and power, the this-sidedness and worldliness of their thinking. But because the word (language) is common to us, the thought of unicorns becomes a shared reality.

Feuerbach, although recognizing the role of the collective in the constitution of knowledge, nevertheless theorizes the individual as the seat of its essence; he thinks the essence of a person as his/her own essence. As a result, 'consciousness is self-action, self-confirmation, self-love, – self-love not in the sense of the animalistic – pleasure in its own perfection. Consciousness is the characteristic mark of a perfect being'.[68] This is not unlike the fundamental constructivist (enactivist) thesis that human beings are the result of their own constructions, as end products of their own actions bearing the same characteristics as the actions. According to Feuerbach, the *nature* of human beings is determined by the essence of the species, which is the *absolute essence* of the individual. The Sixth Thesis on Feuerbach states however that 'human nature is not an abstractum inherent in the single individual. In its reality it is the ensemble of societal relations'.[69] In the fragmentary text entitled 'Concrete Human Psychology', Vygotsky paraphrases this dictum comes in this way: 'the *psychological* nature of man is the totality of [societal] relations shifted to the inner sphere and having become functions of the personality and forms of its structure'.[70]

Society is produced by, and produces, the ensemble of human relations. 'Where a relation exists', Marx and Engels write, 'it exists for me, the animal does not "*re-*

[67] Vygotsky, 'Concrete Human Psychology', 59.
[68] Ludwig Feuerbach, *Das Wesen des Christenthums* (Leipzig: Otto Wigand, 1841), 9.
[69] Marx and Engels, *Werke Band 3,* 6.
[70] Vygotsky, 'Concrete Human Psychology', 59.

late" itself to anything, does not "*relate*" itself at all. For the animal its relation to others does not exist as a relation'.[71] To exist for me, the relation (re-) appears as such in consciousness. This idea is integral to the role of relations in Vygotsky's approach, and he therefore takes any higher psychological function to have been relations with others before; that is, before becoming a psychological function, a function first *was* a real, social relation between two concrete human beings. Vygotsky does not state that there was something *in* the relation that then was internalized. Instead, the function itself was at one point a relation between people. In this approach, speech is paradigmatic, for it is both a social relation and a psychological means. In the case of speech, the connection between social relation and psychological function is stated even more strongly: 'All forms of verbal communication between adult and child later become psychological functions'.[72] Thus, whereas the formulation 'any higher function was the social relation between people' does not imply that all social relations will be higher functions, this latter formulation describes development in the case of verbal communication. The important role of social relations leads Vygotsky to take up some ideas from another Marxian psychologist and philosopher working in France, Georges Politzer, who conceived of a *psychology in terms of drama*.

Vygotsky was impressed but never really took up the conceptualization of drama, though a recent commentator has made a strong case that this idea was indeed underlying the psychologist's approach.[73] Drama takes into account the whole life as it plays itself out in particular situation, for example, where there are relations between people. In a relation, one person responds to another so that the origin of her thinking does not lie in herself but comes from the outside, from what affects her. The end point of her action does not lie in her, but the action – something she does or says – also is on the outside, another person or thing. This is a fundamentally Spinozist conception. The same is the case for the other person, who responds to the person responding to him. Doing and talking takes into account the other, who has acted before and who will be the recipient. Any saying and doing – points to be developed later in this book – therefore begins and ends on the outside of the person in relation with another. This is why drama is neither internal nor external: it always has front and back stage, and neither can be understood without the other. Drama therefore constitutes the locus of development, and the contradictions within the unfolding drama are a manifestation of both movement and force within the dramatic event. Thinking the human psyche in terms of drama takes psychology away from the traditional foci: that of scientific psychology, which seeks its object in the *world of nature*, and that of interpretive psychology, which seeks its object in the *world of the mind*. Vygotsky's approach, therefore, is consistent with the analyses that appear in *The German Ideology*, which orients its readers to the whole person, whose life determines her consciousness. 'Once the object of psy-

[71] Marx and Engels, *Werke Band 3*, 30.

[72] Vygotsky, 'Concrete Human Psychology', 58.

[73] Nikolai Veresov, 'Zone of proximal development (ZPD): The hidden dimension?' In *Språk som kultur – betydningar i tid och rum,* edited by Anna-Lena Østern and Ria Heilä-Ylikallio (Vasa: Åbo Akademi, 2004), 13–30.

chology is defined as drama, the totality of the individual becomes the initial and fundamental hypothesis, without which no fact and no notion of psychology is conceivable, and the elementary analysis becomes not only possible but also really fecund'.[74] As a result, concrete psychology tends toward its parts that are dramatic in themselves. That is, when concrete psychology decomposes the drama, it does so on the assumption of the totality of the individual cannot be reduced, in the same way as facts that reveal themselves in the decomposition. A dramatic conception of mathematics is developed in chapter 9.

On Method

There are considerable implications for doing research when we take Vygotsky's theoretical approach. In this section, some of these implications are developed around the notions of the primacy of praxis, unit analysis, and dialectics and the implications thereof for doing 'concrete human psychology'.

Primacy of Praxis

We may distinguish *praxis*, the actual, sensual labor through which things get done from practice, the *conception* of the customary ways in which humans do things as it appears in the minds of people. The distinction is the same that Marx makes in denouncing all preceding materialisms, which capture sensuality only in the form of the *object or intuition*; but no as *sensual human activity, praxis*; not subjective'.[75] Concrete human psychology is concerned with how real living people, in the here and now of their concrete lives, make, and are made by, the everyday world that they inhabit; concrete human psychology attempts to understand how people succeed in doing what they do without having a conception or representation thereof in their minds. Thus, if we see a red light, we stop at the intersection. We do not see the light, interpret it, and then direct the relevant organ to push or pull the brakes. It is this orientation that we find in the writings of the late Vygotsky, who directs us to 'identify the needs, interests, incentives and tendencies that direct the movement of thought' rather than divorcing thinking 'from the full vitality of life'.[76] To do so, we need to orient toward praxis as the starting point of our research. That is, there ought to be a primacy of praxis when we attempt to understand mathematical thinking, learning, and development.

[74] Politzer, 'Les fondements', 59–60.
[75] Marx and Engels, *Werke Band 3*, 5.
[76] Vygotsky, 'Thinking and Speech', 50.

Above I note the Marxian realization that consciousness is conscious being. First there is once-occurrent, always novel and in part unexpected being[77]; and then consciousness thereof follows. As a consequence, 'it is not consciousness that determines life, but life that determines consciousness'.[78] Marx and Engels distinguish between the two ways of proceeding. The first position begins with the consciousness as the living individual, whereas with the second perspective, analyses begin with real practical life, real, sensuous living human beings and then consider consciousness only as the consciousness of these beings. That is, concrete psychology needs to investigate psychological facts in terms of the segments of a drama, each segment itself dramatic through and through; and each act always implies the individual considered *as a whole*. In this way, social scientists can stop speculating and begin doing positive science that begins with real, sensuous life. This science is concerned with representing what people practically do, which leads to practical processes of human development. Scientists can stop stating empty phrases about consciousness, replacing these by real knowledge. For example, rather than pronouncing themselves about knowledge and knowledge construction, scientists would investigate real participation in human activity to learn about the ways in which individuals make thinking available for each other in the public forum and the ways in which order is produced *for the purpose to be seen as such by others*. When people queue, they are not just placing themselves in an already existing structure based on some mental schema but they are in fact acting *for others to see* them as lining up rather than doing something else. When individuals do something as mathematicians, then they act in ways so that others see them do *mathematics* rather than something else. That is, *anything that we recognize as mathematical is observably displayed in a public arena; thus, mathematics is out there in the open and we do not have to get into the head*. It is only when we force students to work on their own that mathematics *appears to be individual*, when in fact it still is social given that it *was* a social relation first. Students in school, especially those experiencing difficulties, may have to be reminded that they have to exhibit the mathematics of their mathematics, for example, when in the exam instructions requesting them: 'show your work'.

For Vygotsky, it is apparent that psychology needs to follow the route that Marx and Engels have outlined for the procedure of history. Contrary to idealist history, which tends to search for categories in every period, a Marxian conception of history 'remains constantly on the real *ground* of history, does not explain praxis out of ideas, explains formation of ideas out of material praxis'.[79] Thus, contrary to idealist psychology, concrete psychology does not seek recourse to 'meanings' to explain social life and how individuals contribute to its production, but uses the unfolding drama of social life to explain whatever consciousness arises.

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[77] Mikhail M. Bakhtin, *Toward a Philosophy of the Act* (Austin: University of Texas Press, 1993); also, '<K filosofii postupka>', in *Sobranie socinenij t.1* (Moscow: Izdatel'stvo russkie slovari jazyki slavjanskoj kul'tury, 2003).

[78] Marx and Engels, *Werke Band 3*, 27.

[79] Marx and Engels, *Werke Band 3*, 38.

The fundamental problem of classical psychology in both of its versions, and, with it, in classical educational research, is that the psychological facts are derived from the transpositions of life into the world of the text. Such a transposition destroys the drama of the drama. Thus, interpretive psychology and interpretive research more generally take as their object the lexical words that a person produces and derive from it their conjectures about the contents, structures, and processes of the mind. But all of this work is concerned with objects that appear in the world of text writ large, which no longer is the dialogical and transactional world that we inhabit, a world in which there is no time out (to think), but a world in which actions are ascribed to individual actors and the constitutive social relations have been reduced to *self*-action or *inter*-action.[80] Thus, to do *concrete* psychology, we have to return to the drama of social life, follow events as they unfold and investigate what people make available to each other that is taken up, transformed, and returned as part of the response.

Both Vygotsky and Politzer consider *geisteswissenschaftliche* (i.e. interpretive) psychology to be as reductionist as scientific psychology, even though the former tends toward understanding life as drama. Its problem arises from the fact that interpretive psychology is concerned with idealist 'meanings', psychological facts that result from the *mind*. Politzer is critical of the reliance on 'meaning' in interpretive psychology, which leads it to a primacy of the mind. Unsurprisingly, he uses the term 'mythological psychology' to denote this approach to psychology, which itself is opposed to 'scientific psychology' that only studies physiological and other bodily reactions. Concrete psychology, for its part, is concerned with the organization of how *praktische Menschenkenntnis* [practical ability to judge character] operates. It is therefore not unlike ethnomethodology, concerned with the ways in which everyday people not only produce the orderliness of their lives but also do so in ways that the order and the making thereof are available to other participants, who, in these expressions, recognize their own orderly, order-producing ways. To understand human beings, research needs to focus on the real societal relations, the real life conditions that humans produce and that produce them.

Marx and Engels note that there is a significant instant when the division of labor becomes such that material and mental labor are separated. At this point, 'from this moment onwards consciousness *can* really flatter itself that it is something other than consciousness of existing praxis, that it *really* represents something without representing something real'.[81] Before that point, the production of ideas, thoughts, representations, consciousness, and the likes is enmeshed with practical material activity and the material exchanges of real people – that is, it is enmeshed with the language of real life.

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[80] Ricœur shows how there are three forms of mimesis operating: the first, when we make and observe an order while participating in social life; the second, when social life is transposed into the world of text; and the third, when the world of text is transposed back into social life as part of the implications and applications of research results. Paul Ricœur, *Time and Narrative I* (Evanston, IL: Northwestern University Press, 1984).

[81] Marx and Engels, *Werke Band 3*, 31.

Theory and method are intimately related; they have to be related or there will be contradictions in the empirical results. Thus, if theory conceives of categories that are the smallest unit of thought that make sense, then research methods need to be devised such that they maintain the integrity of the analytic units chosen. If, for example societal activity is chosen as the minimal unit – as in modern, third-generation articulations of cultural-historical activity theory – then we cannot investigate mathematical behavior while being oblivious to schooling as the relevant unit that retains all the characteristics of society. We must not consider mathematical thinking apart from the fact that whatever a child or student says in a think-aloud or clinical interview session is part of an event organized for the specific purpose of collecting data concerning mathematical thinking. Similarly, if our research is to investigate moving phenomena, such as learning or development, then the method needs to take into account of the movement and not articulate a structural theory where entities remain stable and are changed only from without. Vygotsky, as Spinozist-Marxian thinkers generally, is interested in a dynamic account, which, inherently, has to be plural (multiplicious), for the phenomenon is changing *within* the unit chosen and, therefore, is non-self-identical. Finally, a theory that acknowledges societal relations as the essence of what it is to be human requires data and forms of analysis that do not reduce behavior to the individual but shows where anything apparently individual first has originated in a collective behavior. The following subsections elaborate on each of these constraints and affordances of method.

Unit Analysis

> 'Those things that are common to all things and are equally in the part as in the whole can be conceived only adequately'.[82]

Vygotsky charges traditional psychology with something that the science has taken over from philosophy: the separation of thinking from being. Thinking, rather than being a mode of acting in the world, thereby came to be transformed into an autonomous stream of thoughts. As evidenced in much of (social) constructivist theorizing, thinking itself is the thinker of thoughts rather than being in the service of a human individual while acting in the world. In his 'The Historical Sense of the Crisis', Vygotsky quotes from Feuerbach's text against the mind–body dualism, according to which 'the difference between *thinking* and *being* is not sublated in psychology. Even in regard to thinking one has to distinguish between the thinking of thinking and the thinking in itself'.[83]

The problem also exists when investigations start on the other end, with the body, when psychologists, as good natural scientist, focus on the ready-made things in the way that these appear to them in their present existence. 'Only then

[82] Spinoza, 'Ethics', 265.
[83] Feuerbach, *Werke, Zweiter Band*, 353.

can the question arise: "How can spiritual, ideal thought emerge directly from the spatial interaction of soulless material bodies?"'.[84] The problem is eliminated as soon as there is a universal substance, Nature, which constitutes an integrality, which is the general foundation of the Extension–Thought unity. In the Spinozist take, theorizing does not begin with the parts (elements) but with the whole because the aim is 'to understand the place and role of every part, "mode", in the general plan of the whole, to understand the par as a separate *manifestation* of universal, integral Nature. The whole, in Spinoza's philosophy, is more than the sum of its parts, because it is present wholly in each one of its parts and creates the parts that are lacking'.[85] The late Vygotsky notes in one of the last texts he wrote before his death, the introduction to *Thinking and Speech*: 'a psychology that decomposes verbal thinking into its elements in an attempt to explain its characteristics will search in vain for the unity that is characteristic of the whole. These characteristics are inherent in the phenomenon only as a whole. When the whole is analyzed into its elements, these characteristics evaporate' because 'the internal relationships of the unified whole are replaced with external mechanical relationships between two heterogeneous processes'.[86] Thus, a psychology that begins with the elements inherently cannot get to the 'more than the parts' other than by introducing *external* forms that produce the relations between the parts. In the case of body and mind, Descartes introduced the pineal gland; Hegel trickily constructed a process of *sublation*; and modern-day psychology and mathematics education introduce tools and signs to function as the mediators between the two. But, Vygotsky charges, though united in the sign, the parts are theorized to coexist, but completely isolated one from the other. The same external mediators are required to bring together two externally related subjectivities in the pursuit of a common reality, intersubjectivity (see chapter 5). Vygotsky is interested in the 'concrete characteristics of the whole', which alone can explain the function of thinking and speech. Thus, an analysis needs to begin with a unified whole, such as the intersubjectivity in a social situation, to understand its parts, such as any individual or intrasubjectivity that arises and is expressed in that situation.

The problem of method is nowhere better characterized for Vygotsky than when researchers 'smash' the unity of the physical and semantic parts of speech, that is, when research splits sound (sensible) from signification (supersensible). Readers clearly will be able to identify the massive amount of studies that focuses on signification ('meaning') while disregarding sound. When students' or teachers' bodies enter the analyses, then mediators, such as schemas, are required to make a connection between the movement of a hand and arm with the words that are taken to express 'their' 'meanings'. In this way, however, only an abstract unity is achieved, literally manifested in the abstraction that a schema represents. But, as Vygotsky notes, 'divorced from thought, sound loses all the unique features that are characteristic of it as the sound of human speech, the characteristics that distin-

[84] Felix T. Mikhailov, *The Riddle of the Self* (Moscow: Progress Publishers, 1980), 66.
[85] Mikhailov, *Riddle*, 67.
[86] Vygotsky, 'Thinking and Speech', 45.

guish it from the other types of sound that exist in nature. … As a consequence, this research has not been able to explain why sound possessing certain physical and [psychological] characteristics is present in human speech or how it functions as a component of speech'.[87]

Vygotsky proposes to study human behavior in terms of 'units', which are the results of analysis that retain all the properties of the whole. Years before he articulates this aspect of his method in *Thinking and Speech*, he already encountered it in the Politzer text as presented above. Vygotsky thus proposes one such unit, the inner aspect of the word, its 'signification [značenie]' often translated as 'meaning'. The problem with the English 'meaning' is that it concerns the semantic aspect of the word alone. But such a merging must be possible in other languages as well, for Vygotsky points out that in most research, only the external, semantic side is studied. 'Word meaning', however, pertains to the *inner* aspect of the sound-word, which includes all features of the whole situation. As a result of focusing on external aspects, 'word meaning has been dissolved in the sea of all representations in our consciousness or our thinking'. Inherently, 'meaning', conceived as suprasensible in nature, therefore cannot be the unifying category. It cannot be, speaking with Spinoza, an adequate concept unless it is enlarged to encompass the semantic aspects as one of its modes. Thus, Vygotsky himself points out that značenie ['meaning'] is in excess of the semiotic aspect so that 'the variation of meaning is a deeper, more essential, more internal analysis of a semiotic operation'.[88]

In the Spinozist take characterizing Vygotsky's thoughts near the end of his life, that whole has to be such that it has sound and semantic aspects as its modes. A perhaps more fruitful way of proceeding is to focus on the function or value of the sound-word in the situation as a whole, an approach consistent with the Russian 'značenie', which also translates as 'value'. In the notes closer to his death, after reading *The German Ideology* and the prominence of consciousness, Vygotsky comes to orient toward *sense*. Thus, in 'documents from 1932, Vygotsky points out that the child is oriented toward sense virtually from the very first days of her life; her first questions are about sense rather than the meanings of her surroundings'.[89] He is interested in the complex dynamics of sense and how it manifests itself in thinking and speech. That sense has both visible and invisible aspects. Without the visible aspects, movement is incomprehensible. But there is more to sense because, in the case of movements, they exceed what is necessary and, therefore, are never precise. As a result, 'movement always has a latent, inner sense of movement, which always expresses the person's attitude to the goal, internal struggle, hesitation, additional goal, latent tendency or motivation, hot temper, weakness, exaggeration of the goal, attainment of the goal for show'.[90] Any movement may there-

[87] Vygotsky, 'Thinking and Speech', 46.

[88] Lev S. Vygotsky, in Ekaterina Iu. Zavershneva, 'The Vygotsky Family Archive: New Findings – Notebooks, Notes, and Scientific Journals of L.S. Vygotsky (1912–1934)', *Journal of Russian and East European Psychology* 48(1), 55.

[89] Zavershneva, 'Notebooks', 44.

[90] Lev S. Vygotsky as quoted in Zavershneva, 'Notebooks', 53.

fore be in excess of (more than) what is required by the situation; but it also may be less than what is necessary. Vygotsky sees in this 'more or less' the key to the latent, invisible part of sense.

Dialectics

Vygotsky pursues an agenda of *dialectical psychology*, which 'proceeds first of all from the unity of mental and physiological processes'. He refers to Spinoza when stating that in dialectical psychology, mind is not situated outside nature, as it appears in Descartes' work, but is 'a part of nature itself, directly linked to the functions of the higher organized matter of our brain'.[91]

In his analysis of the historical sense of the crisis in psychology, a method-related and methodological investigation, the earlier Vygotsky relates many of the problems of psychology to method. He considers dialectics the most general, therefore most universal science, because it covers the universe in the way we know it, including nature, culture, history, and thinking. But he suggests that the principles of dialectics must not be imposed on the phenomena of interest from the outside. Instead, 'we must reveal the *essence* of the given area of phenomena, the laws of their change, their qualitative and quantitative characteristics, their causality, we must create categories and concepts appropriate to it'.[92] That is, Vygotsky argues for a data-based investigation of the phenomena and to show, through analysis, any dialectical principles at work. This is consistent with, and arises from the dialectical method, which insists on explaining the world from the world itself – a data-driven science that develops theory rather than a science that begins with theory and imposes it on the world – while leaving, if science is not yet advanced enough, the justification to the future. Thus, categories and concepts have to be appropriate to the phenomena, developed out of rather than foisted upon them. He alludes to the text of Engel's text on dialectics, which states that the idealists commit a serious mistake, which 'lies in the fact that these laws are foisted on nature and history as laws of thought, and not deduced from them'.[93] The result of this mistake is a world ordered according to categories and concepts, theory over praxis, rather than a set of categories and concepts that stand for ideas that are adequate to the natural phenomena in the Spinozist-Marxian sense. A second mistake coincides with the first, according to which the investigators do not realize the historical nature of the categories and concepts that they use.

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[91] Lev S. Vygotsky, 'Mind, Consciousness, the Unconscious', in *Lev S. Vygotsky Collected Works: Volume 3 – Problems of the Theory and History of Psychology* (New York: Springer, 1997), 112.

[92] Lev S. Vygotsky, 'The Historical Meaning of the Crisis in Psychology: A Methodological Investigation', in *The Collected Works of L. S. Vygotsky, Volume 3, Problems of the Theory and History of Psychology* (New York: Springer, 1997), 330.

[93] Friedrich Engels, 'Dialektik', in *Werke Band 20* by Karl Marx and Friedrich Engels (Berlin: Dietz, 1975), 348.

Engels articulates three main laws of dialectics, which are important for understanding how the earlier Vygotsky proceeds and what he articulates about psychological development: (a) the law of the transformation of quantity into quality and vice versa; (b) the law of the interpenetration of opposites; and (c) the law of the negation of the negation. Although Engels, following Hegel, uses natural (material) phenomena to exemplify this law, he suggests that it is equally valid for the history of human society and for living bodies, though it operates under complex conditions so that the quantitative relations are much more difficult to exhibit. These laws are highly appropriate and particularly useful in the context of Vygotsky's earlier agenda to address the psychophysical problem, that is, to overcome the apparent abyss between mind and body. This so because the second law concerns the interpenetration of opposites, here mind and body, which is achieved precisely by means of the Spinozist move to the one substance that has both modes as its attributes. The negation of mind by body, and of body by mind, thereby is negated. The first law would be of particular importance for describing developmental phenomena, where qualitatively new functions – Vygotsky calls these *neoformations* – arise from quantitative changes in old functions.[94] As a consequence of the later Vygotsky, the second law needs to be rethought in terms of a plural singular, where the emphasis is on plurality rather than dualistic oppositions; and the third law needs to be rethought starting with the plural singular, which inherently manifests itself as plurality.

We actually find the same kind of thinking already in Spinoza's texts. This philosopher is concerned with a flaw of a lot of research that exists to the present day, whereby concepts and methods are imposed on phenomena from the outside. Method is defined in advance of the research rather than being developed to be appropriate to the phenomena; and theoretical concepts are imposed on whatever researchers are looking at rather than developing methods and concepts (categories) through a thorough investigation of the phenomena themselves. Spinoza is critical of this approach when he notes that

> 'men are wont to form general ideas both of natural phenomena and of artifacts, and these ideas they regard as models, and they believe that Nature (which they consider does nothing without an end in view) looks to these ideas and holds them before herself as models. So when they see something occurring in Nature at variance with their preconceived ideal of the thing in question, they believe that Nature has then failed or blundered and has left that thing imperfect'.[95]

In attending closely to what people – students and teachers – actually do in mathematics lessons, we can find out not only the order of the world that they inhabit seen through their eyes but also, and more importantly, the methods they use

[94] See Wolff-Michael Roth, '*Neoformation*: A Dialectical Approach to Developmental Change', *Mind, Culture and Activity* (2016); and Wolff-Michael Roth and Alfredo Jornet, *Understanding Educational Psychology: A Late Vygotskian, Spinozist Approach* (Dordrecht, The Netherlands: Springer, 2017).

[95] Spinoza, 'Ethics', 320–321.

for producing and exhibiting this order and its production. It is precisely in their mutual constitution within the system as a whole – which, in the dialectics of nature, is the Spinozist substance that is *causa sui* – that we understand social phenomena, because 'the transaction is the true *causa finalis* of things'.[96] Thus, it is likely that other social sciences are in the same boat as sociology, where, as Pierre Bourdieu suggests, researchers tend to chose as objects of research problems that involve the social order, discourses, and ways of thinking and ordering the world *without investigating the origin of the object, problems, and discourses*. Rather than imposing theoretical concepts and methods – whether these derive them from the everyday world or from the specific discourses of the scientific community – researchers need to be aware not only of the preconstructed concepts, which surround them everywhere, but also the tools of the construction. Investigative endeavors, therefore, first and foremost need to investigate the pre-constructed objects, that is, their own discourses, interests, and the historical processes of the construction of the former. If investigators do not do so, then the research risks '*borrow[ing] its problems, its concepts, and its instruments of knowledge from the social world*', and then records as data, facts, or representations that 'are the *product of a prior stage of science*. In short, it records itself without recognizing itself'.[97] The same task is posited by a cultural-historical psychology of the type that Vygotsky envisioned. The research reconstructs the present, which initially appears in thought in a superficial way, in terms of pre-constructed, generally disconnected determinations. It is at that point that we enact the most radical break from the formal methods that impose special order-producing (scientific) methods. Not surprisingly, therefore, Vygotsky tends to investigate the history of the problems that he chose, such as when reviewing 'The Teaching of Emotions', which was to become the preparation for his work on a Spinozist-Marxian take on affect.

Researchers need to constantly attend 'to the details of the research procedure', which 'should have the effect of putting you on notice against the fetishism of concepts, and of "theory", born of the propensity to consider "theoretical" instruments … in themselves and for themselves, rather than to put them in motion and to make them *work*'.[98] A typical move in a cultural-historical approach – which Vygotsky held to be the 'maximal universal science' – is to return to the most general 'cell form' of the phenomenon and develop it together with the associated, adequate category. In Spinoza's words, this means that 'in examining natural phenomena we first of all try to discover those features that are most universal and common to the whole of Nature, to wit, motion-and-rest and the rules and laws governing them which Nature always observes and through which she constantly acts; and then we advance gradually from these to other less universal features'.[99] Given that Nature continuously evolves, this inherently requires a historical approach, which is rec-

[96] Friedrich Engels, 'Notizen und Fragmente', in *Werke 20* by Karl Marx and Friedrich Engels (Berlin: Dietz, 1975), 499. *Causa sui* is 'cause of itself', whereas *causal finalis* translates as 'final cause'.

[97] Bourdieu, 'The Paris Workshop', 236.

[98] Bourdieu, 'The Paris Workshop', 228.

[99] Baruch Spinoza, 'Theological-Political Treatise', in *Complete Works* (Indianapolis: Hackett, 2002), 460.

ognized to be the chosen method to any scientific problem, among others because it allows us to understand how the situation of interest and associated discourses came to be in the first place. It is in this light that we can make sense of Vygotsky's endeavor to identify the genetic roots of the problem he investigates, whether it was the relationship between thinking and speech or the problem of the emotions. This would allow him to begin with the internal unity of a phenomenon that has thinking and speech as two of its modes. He thus begins with positing a complex whole, a 'dynamic meaningful system that constitutes *a unity of affective and intellectual processes*'.[100]

On Methods for a 'Concrete' 'Human' Psychology of Mathematics

With respect to materialist philosophy, Marx, all the while taking on its achievements, also critiques it as being inconsistent with their content. 'Even in the case of philosophers', he writes, 'who give systematic form to their work, Spinoza f[or] i[nstance], the true inner structure of the system is quite unlike the form in which it was consciously presented by him'.[101] Thus, 'the chief defect of all previous materialisms (including Feuerbach's) is that the object, reality, sensibility, is conceived only in the form of the *object or of intuition* but not as *sensual human activity, praxis,* not subjectively'.[102] The result of this is that the active aspect of activity is developed by idealist philosophers, who do not conceive of the real, sensual labor of people but instead approach it as the result of dealing with the objects in thought and then acting on them in the world. The kind of psychology Vygotsky envisions is a *concrete human psychology*, where the 'concrete' and 'human' are to be taken literally, in the sense in which we encounter everyday life. It is for this reason that he focuses on sociogenesis and the articulation of higher psychological functions that first appear *as* collective behavior and then become individual, no less social behavior. Experimentally, then, this individual behavior can be unfolded into collective behavior – not unlike what in more recent years has been realized by means of breaching experiments or the study of the social in 'breakdown' situations. Fundamentally, the invisible labor underlying the order and rationality of everyday human behavior becomes visible in problematic situations where what we normally do and assume no longer works. Members to any setting then exhibit the orderly and order-producing work for each other so that they can subsequently return to the normal way of going about life. This sort of research no longer deals with representations, mental frameworks, or meanings but with the real, sensuous labor of people, who are subject and subjected to the order that they themselves produce visibly for others as much as for themselves.

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[100] Vygotsky, 'Thinking and Speech', 50.

[101] Karl Marx, 'Marx an Ferdinand Lassalle – 31. Mai 1858', in *Werke Band 29*, by Karl Marx, Friedrich Engels (Berlin: Dietz, 1978), 561.

[102] Marx and Engels, *Werke Band 3*, 5.

To understand thinking, we need to study its functional relation to the activity as a whole. The relevant social and material order does not just exist out there but is continuously produced and made visible in the ongoing transactional work that also produces the object/motive of the societal activity currently being realized. In situation, people conduct this work together and for each other. Thinking is an integral part of this work, and its study needs to be oriented toward understanding its relations and functions. The methods do not need to be different than those that the people themselves employ for producing and exposing the relevant order of things. Microanalyses, therefore, are not conducted for their own sake, as distinct from the analysis of macro-level social structures. Instead, microanalyses are directed toward the structure of the joint work that produces the visibility of the social. The people doing such work in our studies are understood as the particular staff that produces the social phenomenon of interest. In this approach, there no longer is a distinction between method and the system, for research reveals the structure of the work that produces the phenomena that other research presupposes as a fact.

We may investigate human in a concrete manner behavior generally and mathematical cognition specifically by focusing on whole persons, inherently involved in an aspect that is part of the fullness of their lives. Vygotsky finds appealing the idea that we experience life as drama and in dramatic ways. Rather than investigating thought and thinking through the study of 'conceptions', 'mental frameworks', 'beliefs', 'meanings', 'interests', 'motivations', and the likes we are better off investigating how people, in flesh and blood, conduct everyday life such that it can be seen as reasoned and rational. For this, we actually have to pursue what people really in life do rather than what they are made to do and reason in events transposed to the level of the text. To elaborate this distinction, it helps to invoke the concept of *mimesis* that allows distinguishing between different forms of text.[103]

Everyday human behavior is patterned even when the 'representations' and verbal 'meanings' are absent, as can be seen in the ways we move and do certain things. But we also talk; and this talk has as its first and foremost function to let any order be seen and to orient toward its orderly production. That language, therefore, is part of the everyday world but also (sometimes) makes it present again, that is, it is a copy of the world. We may denote this form of imitation of actual practice that occurs in praxis as $mimesis_1$. In most research, this order and its achievement is not of interest, but it is precisely the point of ethnomethodology, which constitutes a radical alternate to all other forms of research, qualitative and quantitative research combined. This radical nature of the alternate can be understood as soon as it is recognized that interpretive and scientific psychology (of mathematics education) do not investigate and operate on the real worldly phenomena but on a transposition thereof that exists in narrative and representational forms. Whatever the method for producing and identifying the order in terms of scientific concepts, they are different from what ordinary people do. This is why scientists have to state the methods they use; and this is why the methods sections take such an important

[103] The following exposition of $mimesis_1$, $mimesis_2$, and $mimesis_3$ is derived from chapter 3 of Paul Ricœur, *Time ad Narrative Vol. 1* (Evanston: Northwestern University Press, 1984), 52–87.

position in quantitative as much as in qualitative scientific research articles. On the other hand, if the methods of identifying and producing order are not different from what we do in the everyday world then there is no need to specify them.

The order in the narratives – which may be in terms of 'meanings', 'positioning', or 'discourse' of qualitative studies or the reaction times, variables and measures, and latent constructs so dear to quantitative studies – then is a second form of mimesis referred to as *mimesis*$_2$, the mimesis characteristic of a created world, the world of the text. We then see that mimesis$_2$ captures the world of theory, where things work on paper, constituting abstractions from and of the real world that we inhabit and the order of which is made available as part of our lives in and as mimesis$_1$. This gap was precisely a key issue in the work of Marx, 'who avoided introducing as an explanatory principle abstractions which had no empirical functions, and which could not be vindicated by observing the ways in which human beings actually behaved'.[104]

Finally, mimesis$_3$ refers to the way in which our everyday world comes to be reconfigured by the narrative productions in the textual world of mimesis$_2$ (theory). This is where we may locate the complaints of practitioners, which are concerned with the 'irrelevance' of theory to practice. Readers will be familiar with such complaints from teachers, who often state that what they learn in university courses is of little use in their actual praxis; and what their praxis is little if at all captured in the theories that they encounter in their university courses. The situation is similar in the case of other fields, such as when apprentice electricians encounter one kind of mathematics in their college courses – e.g. the trigonometry required for calculating the geometry of electrical conduits with paper, pencil, and calculator – and on the job site, where they use tools engraved with markers and numbers that allow them to bend real conduit in much less error-prone ways.[105]

In this approach, mimesis$_2$, 'the configurating operation constitutive of emplotment is a result of its intermediary position between the two operations I am calling mimesis$_1$ and mimesis$_3$, which constitutes the two sides [*l'amont et l'aval*] of mimesis$_2$'.[106] In the original, Ricœur uses the terms 'l'amont', upstream, and 'l'aval', downstream, which better than 'the two sides' that appears in the translation gives the sense of the nature and relation of the three ways in which – and types of situations where – order is produced. The distinction between the three loci of order production allows us to understand our own research endeavors. In my own praxis (analysis), manifested throughout the chapters that follow, I exhibit the practical order production and practical exhibition of order and ordering work in the world of the participants – i.e., I am concerned with mimesis$_1$ – rather than drawing on special methods to translate the participant world into a world of text to create mimesis$_2$ and then to analyze the structures within mimesis$_2$.

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[104] Sidney Hook, *From Hegel to Marx: Studies in the Intellectual Development of Karl Marx* (New York: Columbia University Press, 1994), 277.

[105] Wolff-Michael Roth, 'Rules of Bending, Bending the Rules: The Geometry of Conduit Bending in College and the Workplace', *Educational Studies in Mathematics* 86 (2014): 177–192.

[106] Ricœur, *Time ad Narrative*, 53.

Because we experience life as drama, Politzer and Vygotsky suggest developing psychology in terms of drama. If any higher psychological function is a relation with another person *first*, then the object of the psychologist no longer is an abstraction but the real relation of two people. This relation, as any human relation, can be expressed in dramatic form and in terms of recognizable roles – e.g. parent–child, child–teacher, child–peer, etc. Rather than establishing mythologies, psychology would then become entirely 'concrete', dealing in and with the kinds of things, phenomena, and concepts that are from everyday life; and it would be completely 'human', for drama is *essentially* human. It may not surprise that for Aristotle drama (tragedy and comedy) played such an important role, for this literary form takes its central concerns from real life, depicts them in real life forms, and returns them to real life in recognizable form when audiences appreciate recognizing in and learning from represented life, their real life, which they now better understand and for which they learn in turn.