A MATHEMATICS TEACHER'S APOLOGY [1]

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*A comment after reading 'Talking about subject-specific pedagogy', **25**(3)*: This discussion immediately caught my attention after having recently participated in an interdisciplinary course on learning strategies in Norway.

While the course was to be independent of subjects taught, the reception among the teachers was not. Mathematics teachers were on the whole less enthusiastic and reported more often than others that they had not implemented the promoted techniques in their classes since the previous session. The language teachers and social science teachers, of course, assumed that the mathematics teachers (as usual) "don't care". This is unfair. Many of us are deeply concerned about strategies for learning mathematics, but the recommended methods somehow do not fit our needs. On the other hand, being mathematics teachers, we did not argue our case very well, and that is why I write this letter. I want to share a picture I tend to draw for myself.

A common approach in teaching in Norway is to consider the object to be learned as "a certain area". This wording is to emphasize the existence of "horizontal" relations to neighbouring knowledge as a means to convey meaning. *Surveying* is then a natural and tentative starting point, and, even at a premature stage, attempts to organise the material will both motivate the students and promote their learning. It is my impression that a lot of popular learning strategies are based on this reasonable way of seeing things. But does this picture apply to mathematics? It sometimes does, and sometimes does not.

Mathematical objects are given by their definitions. That is very far from saying that such an object comes with *meaning* as well. Meaning is not a property that belongs to mathematical objects, meaning has to do with our relationship with mathematical objects. Learning about a mathematical object is precisely to gain meaning for mathematical objects.

My thesis is that to gain meaning for a mathematical object, there is no substitute for getting to know *how it works*. Until that is done with some degree of success, the object will be "invisible to the mind's eye", and cannot be organised, nor located in any landscape of knowledge [2] – however crystal clear this position might be in the teacher's mind. Such work on an initially meaningless object can be quite a demanding thing to do, and not always easy to schedule. I therefore picture the process of learning mathematics more as *breakthroughs* [3] on particular *points* than gradually *covering an area*. Organising such breakthroughs into a body of knowledge is *then a finalising* thing to do.

I will therefore elaborate on Dave Hewitt's and Kath Cross's view that a difference of degree is to be found. In my view, the timetable can be said to be *anti-symmetric* in the sense that in science or social sciences, I expect to benefit from starting with the broader picture and *then* go for close-up or in-depth studies, whereas in mathematics the sensible thing for me to do is pretty much the opposite.

It may very well be that we mathematics teachers take the finalising part of the job too lightly. But I think there can be little doubt about the reason why many of us show a lukewarm attitude towards central parts of learning strategies. It is because we consider the *breakthrough* part of the job to be the more important and difficult one [4].

So what might be the characteristics of a *pedagogy for breakthroughs*? One thing that quickly comes to mind is more like an attitude than a particular skill. If I pick one single thing that I would wish my mathematics students to obtain from my teaching, it would be to increase their ability to bear or tolerate *not seeing the solution*. Or rather to realise that *not seeing the solution* to the (mathematics) problem you are facing is a normal stage in the process of doing mathematics.

Is it possible to teach students such a thing? By instruction, surely not. Perhaps by modelling. I read something last summer that I found interesting, from Brown – *What should be the output of mathematical education?* [5] Let me quote without further comment:

> I learned a tremendous amount from my supervisor Michael Barratt. I remember thinking after a long session with Michael: 'Well, if Michael Barratt can try one damn fool thing after another, why can't I?' I have followed this method ever since! [6]

I also have some heretical thoughts about the celebrated meta-perspective in learning. These thoughts are based on particular experiences that I have had with adults trying to learn fairly elementary mathematics, and are consequently perhaps of limited validity. I shall therefore here restrict myself bluntly to stating that bringing students with troublesome learning histories to view themselves as *students learning mathematics*, is often not a useful way to start.

The emotions that follow pictures of oneself as a scared child behind a desk in a huge classroom struggling with quite incomprehensible mathematical signs may help you at the psychologist's, but are not part of productive learning environments. Emotions like that are obstacles that prevent the student from getting in touch with mathematics. If we could bring such students – if only for a while – to forget completely about the meta-perspective that is haunting them, they might get a glimpse of 'the promised land' and subsequently experience motivation of a kind hitherto unknown to them. Whether this is specific to mathematics I do not know.

Notes

[1] With apologies to Hardy who wrote *A mathematician's apology*.
[2] In Kantian terms, knowing a mathematical concept is to be able to "construct it in the intuition". It is this construction, an act (not a thing), which, hopefully, thereafter can find a place in some cognitive structure.
[3] My notion *breakthrough*, as an undefined term, is to be understood naturally.
[4] I suppose Fermi made the same judgement when he replied to the student who asked for an account of elementary particles, "If I could remember the names of all these particles, I'd be a botanist." The kicks are from the breakthroughs, not the bookkeeping.
[5] Sierpinska, A. and Kilpatrick, J. (1998) *Mathematics education as a research domain: a search for identity*, Dordrecht, The Netherlands, Kluwer Academic Publishers, p. 468.
[6] It is hard to image Barratt (and Brown) spending "long sessions" doing "one damn fool thing after another" on *different tasks* – they're obviously struggling with one single problem. The situation then fits nicely to the notion of breakthrough.